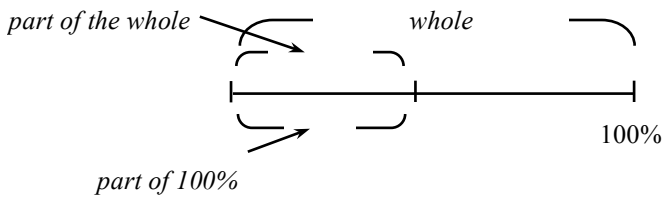A variety of percent problems described in words involve the relationship between "the percent," "the part" and "the whole." When this is represented using a number line, solutions may be found using logical reasoning or equivalent fractions (proportions). These linear models might look like the diagram at right.
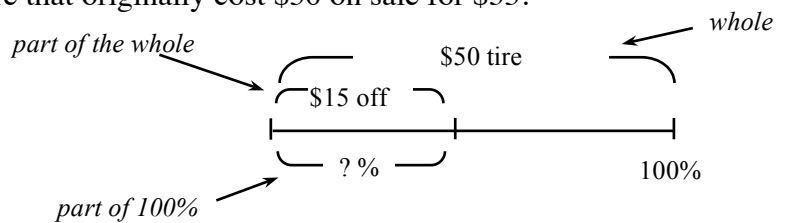

For additional information, see the Math Notes box in Lesson 5.1.2 of the *Core Connections, Course 2* text.

Example 1

Sam's Discount Tires advertises a tire that originally cost $50 on sale for $35. What is the percent discount?

A possible diagram for this situation is shown at right:

In this situation it is easy to reason that since the percent number total (100%) is twice the cost number total ($50), the percent number saved is twice the cost number saved and is therefore a 30% discount. The problem could also be solved using a proportion $\frac{15}{50} = \frac{?}{100}$.

Example 2

Martin received 808 votes for mayor of Smallville. If this was 32% of the total votes cast, how many people voted for mayor of Smallville?

A possible diagram for this situation is shown at right:
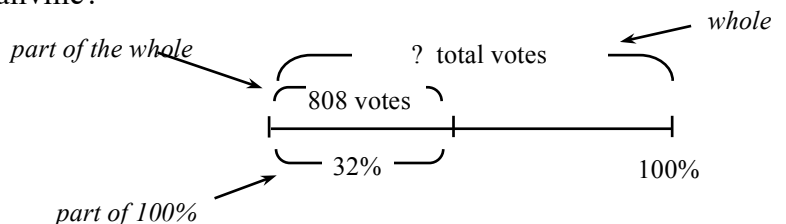
In this case it is better to write a pair of equivalent fractions as a proportion: $\frac{808}{32} = \frac{x}{100}$.

If using the Giant One, the multiplier is $\frac{100}{32} = 3.125$ so $\frac{808}{32} \cdot \frac{3.125}{3.125} = \frac{2525}{100}$.
A total of 2525 people voted for mayor of Smallville.

Note that the proportion in this problem could also be solved using cross-multiplication.

Problems

Use a diagram to solve each of the problems below.

1. Sarah's English test had 90 questions and she got 18 questions wrong. What percent of the questions did she get correct?

2. Cargo pants that regularly sell for $36 are now on sale for 30% off. How much is the discount?

3. The bill for a stay in a hotel was $188 including $15 tax. What percent of the bill was the tax?

4. Alicia got 60 questions correct on her science test. If she received a score of 75%, how many questions were on the test?

5. Basketball shoes are on sale for 22% off. What is the regular price if the sale price is $42?

6. Sergio got 80% on his math test. If he answered 24 questions correctly, how many questions were on the test?

7. A $65 coat is now on sale for $52. What percent discount is given?

8. Ellen bought soccer shorts on sale for $6 off the regular price of $40. What percent did she save?

9. According to school rules, Carol has to convince 60% of her classmates to vote for her in order to be elected class president. There are 32 students in her class. How many students must she convince?

10. A sweater that regularly sold for $52 is now on sale at 30% off. What is the sale price?

11. Jody found an $88 pair of sandals marked 20% off. What is the dollar value of the discount?

12. Ly scored 90% on a test. If he answered 135 questions correctly, how many questions were on the test?

13. By the end of wrestling season, Mighty Max had lost seven pounds and now weighs 128 pounds. What was the percent decrease from his starting weight?

14. George has 245 cards in his baseball card collection. Of these, 85 of the cards are pitchers. What percent of the cards are pitchers?

15. Julio bought soccer shoes at a 35% off sale and saved $42. What was the regular price of the shoes?

Answers

1. 80%	2. $10.80	3. about 8%
4. 80 questions	5. $53.85	6. 30 questions
7. 20%	8. 15%	9. 20 students
10. $36.40	11. $17.60	12. 150 questions
13. about 5%	14. about 35%	15. $120

A ratio is a comparison of two quantities by division. It can be written in several ways:

$$\frac{65 \text{ miles}}{1 \text{ hour}} \text{, } 65 \text{ miles: } 1 \text{ hour, or } 65 \text{ miles to } 1 \text{ hour}$$

For additional information see the Math Notes box in Lesson 5.1.1 of the *Core Connections, Course 2* text.

Example

A bag contains the following marbles: 7 clear, 8 red and 5 blue. The following ratios may be stated:

    a.   Ratio of blue to total number of marbles $\Rightarrow \frac{5}{20} = \frac{1}{4}$.

    b.   Ratio of red to clear $\Rightarrow \frac{8}{7}$.

    c.   Ratio of red to blue $\Rightarrow \frac{8}{5}$.

    d.   Ratio of blue to red $\Rightarrow \frac{5}{8}$.

Problems

1.   Molly's favorite juice drink is made by mixing 3 cups of apple juice, 5 cups of cranberry juice, and 2 cups of ginger ale. State the following ratios:

    a.   Ratio of cranberry juice to apple juice.

    b.   Ratio of ginger ale to apple juice.

    c.   Ratio of ginger ale to finished juice drink (the mixture).

2.   A 40-passenger bus is carrying 20 girls, 16 boys, and 2 teachers on a field trip to the state capital. State the following ratios:

    a.   Ratio of girls to boys.   b.   Ratio of boys to girls.

    c.   Ratio of teachers to students.   d.   Ratio of teachers to passengers.

3.   It is important for Molly (from problem one) to keep the ratios the same when she mixes larger or smaller amounts of the drink. Otherwise, the drink does not taste right. If she needs a total of 30 cups of juice drink, how many cups of each liquid should be used?

4.   If Molly (from problem one) needs 25 cups of juice drink, how many cups of each liquid should be used? Remember that the ratios must stay the same.

Answers

1. a. $\frac{5}{3}$ b. $\frac{2}{3}$ c. $\frac{2}{10} = \frac{1}{5}$

2. a. $\frac{20}{16} = \frac{5}{4}$ b. $\frac{16}{20} = \frac{4}{5}$ c. $\frac{2}{36}$ d. $\frac{2}{38}$

3. 9 c. apple, 15 c. cranberry, 6 c. ginger ale

4. $7\frac{1}{2}$ c. apple, $12\frac{1}{2}$ c. cranberry, 5 c. ginger ale

Two events are **independent** if the outcome of one event does not affect the outcome of the other event. For example, if you draw a card from a standard deck of playing cards but replace it before you draw again, the outcomes of the two draws are independent.

Two events are **dependent** if the outcome of one event affects the outcome of the other event. For example, if you draw a card from a standard deck of playing cards and do not replace it for the next draw, the outcomes of the two draws are dependent.

Example 1

Juan pulled a red card from the deck of regular playing cards. This probability is $\frac{26}{52}$ or $\frac{1}{2}$. He puts the card back into the deck. Will his chance of pulling a red card next time change?

No, his chance of pulling a red card next time will not change, because he replaced the card. There are still 26 red cards out of 52. This is an example of an independent event; his pulling out and replacing a red card does not affect any subsequent selections from the deck.

Example 2

Brett has a bag of 30 multi-colored candies. 15 are red, 6 are blue, 5 are green, 2 are yellow, and 2 are brown. If he pulls out a yellow candy and eats it, does this change his probability of pulling any other candy from the bag?

Yes, this changes the probability, because he now has only 29 candies in the bag and only 1 yellow candy. Originally, his probability of yellow was $\frac{2}{30}$ or $\frac{1}{15}$; it is now $\frac{1}{29}$. Similarly, red was $\frac{15}{30}$ or $\frac{1}{2}$ and now is $\frac{15}{29}$, better than $\frac{1}{2}$. This is an example of a dependent event.

Problems

Decide whether these events are independent or dependent events.

1. Flipping a coin, and then flipping it again.

2. Taking a black 7 out of a deck of cards and not returning it, then taking out another card.

3. Taking a red licorice from a bag and eating it, then taking out another piece of licorice.

Answers

1. independent   2. dependent   3. dependent

COMPOUND PROBABILITY AND COUNTING METHODS   5.2.3 – 5.2.6

COMPOUND PROBABILITY

Sometimes when you are finding a probability, you are interested in either of two outcomes taking place, but not both. For example, you may be interested in drawing a king or a queen from a deck of cards. At other times, you might be interested in one event followed by another event. For example, you might want to roll a one on a number cube and then roll a six. The probabilities of combinations of simple events are called **compound events**.

To find the probability of *either* one event *or* another event that has nothing in common with the first, you can find the probability of each event separately and then add their probabilities. Using the example above of drawing a king or a queen from a deck of cards:

$$P(king) = \tfrac{4}{52} \text{ and } P(queen) = \tfrac{4}{52} \text{ so } P(king \text{ or } queen) = \tfrac{4}{52} + \tfrac{4}{52} = \tfrac{8}{52} = \tfrac{2}{13}$$

For two independent events, to find the probability of *both* one *and* the other event occurring, you can find the probability of each event separately and then multiply their probabilities. Using the example of rolling a one followed by a six on a number cube:

$$P(1) = \tfrac{1}{6} \text{ and } P(6) = \tfrac{1}{6} \text{ so } P(1 \text{ then } 6) = \tfrac{1}{6} \cdot \tfrac{1}{6} = \tfrac{1}{36}$$

Note that you would carry out the same computation if you wanted to know the probability of rolling a one on a green cube, and a six on a red cube, if you rolled both of them at the same time.

Example 1

A spinner is divided into five equal sections numbered 1, 2, 3, 4, and 5. What is the probability of spinning *either* a 2 *or* a 5?
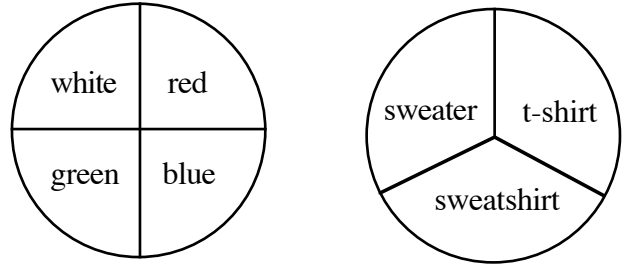
Step 1: Determine both probabilities: $P(2) = \tfrac{1}{5}$ and $P(5) = \tfrac{1}{5}$

Step 2: Since these are *either-or* compound events, add the fractions describing each probability: $\tfrac{1}{5} + \tfrac{1}{5} = \tfrac{2}{5}$

The probability of spinning a 2 or a 5 is $\tfrac{2}{5}$: $P(2 \text{ or } 5) = \tfrac{2}{5}$

Example 2

If each of the regions in each spinner
at right is the same size, what is the
probability of spinning each spinner and
getting a green t-shirt?

Step 1: Determine both possibilities:

$P(green) = \frac{1}{4}$ and $P(t\text{-}shirt) = \frac{1}{3}$

Step 2: Since you are interested in the compound event of *both* green *and* a t-shirt, multiply
both probabilities: $\frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$

The probability of spinning a green t-shirt is $\frac{1}{12}$: $P(green\ t\text{-}shirt) = \frac{1}{12}$

Problems

Assume in each of the problems below that events are independent of each other.

1. One die, numbered 1, 2, 3, 4, 5, and 6, is rolled. What is the probability of rolling *either* a
1 *or* a 6?

2. Mary is playing a game in which she rolls one die and
spins a spinner. What is the probability she will get *both*
the 3 *and* black she needs to win the game?
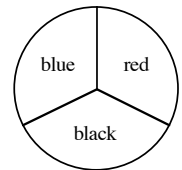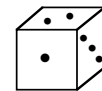
3. A spinner is divided into eight equal sections. The
sections are numbered 1, 2, 3, 4, 5, 6, 7, and 8. What is the probability of spinning a 2, 3,
or a 4?

4. Patty has a box of 12 colored pencils. There are 2 blue, 1 black, 1 gray, 3 red, 2 green,
1 orange, 1 purple, and 1 yellow in the box. Patty closes her eyes and chooses one pencil.
She is hoping to choose a green or a red. What is the probability she will get her wish?

5. Use the spinners at right to tell Paul what his chances are
of getting the silver truck he wants.
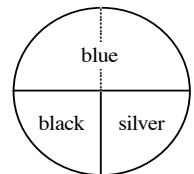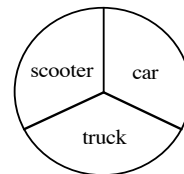
6. On the way to school, the school bus must go through two traffic signals. The first light is
green for 25 seconds out of each minute, and the second light is green for 35 seconds out of
each minute. What is the probability that both lights will be green on the way to school?

7. There are 250 students at South Lake Middle School. 125 enjoy swimming, 50 enjoy skateboarding, and 75 enjoy playing softball. What is the probability a student enjoys all three sports?

8. John has a bag of jellybeans. There are 100 beans in the bag. $\frac{1}{4}$ of the beans are cherry, $\frac{1}{4}$ of the beans are orange, $\frac{1}{4}$ of the beans are licorice, and $\frac{1}{4}$ of the beans are lemon. What is the probability that John will chose one of his favorite flavors, orange, or cherry?

9. A nationwide survey showed that only 4% of children liked eating lima beans. What is the probability that any two children will both like lima beans?

Answers

1. $\frac{2}{6}$ or $\frac{1}{3}$

2. $\frac{1}{18}$

3. $\frac{3}{8}$

4. $\frac{5}{12}$

5. $\frac{1}{12}$

6. $\frac{25}{60} \cdot \frac{35}{60} \approx 0.243$

7. $\frac{125}{250} \cdot \frac{50}{250} \cdot \frac{75}{250} = \frac{3}{100}$

8. $\frac{2}{4}$ or $\frac{1}{2}$

9. $\frac{1}{16}$

COUNTING METHODS

There are several different models you can use to determine all possible outcomes for compound events when *both* one event *and* the other occur: a systematic list, a probability table, and a probability tree. See the Math Notes box in Lesson 5.5.2 of the *Core Connections, Course 2* text for details on these three methods.

Not only can you use a **probability table** to help list all the outcomes, but you can also use it to help you determine probabilities of independent compound events when *both* one event *and* the other occur. For example, the following probability table (sometimes called an **area model**) helps determine the probabilities from Example 2 above:

	$\frac{1}{4}$ white	$\frac{1}{4}$ red	$\frac{1}{4}$ blue	$\frac{1}{4}$ green
$\frac{1}{3}$ sweater				
$\frac{1}{3}$ sweatshirt				
$\frac{1}{3}$ t-shirt				■

Each box in the rectangle represents the compound event of *both* a color *and* the type of clothing (sweater, sweatshirt, or t-shirt). The area of each box represents the probability of getting each combination. For example, the shaded region represents the probability of getting a green t-shirt: $\frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$.

Example 3

At a class picnic Will and Jeff were playing a game where they would shoot a free throw and then flip a coin. Each boy only makes one free throw out of three attempts. Use a probability table (area model) to find the probability that one of the boys makes a free throw, and then flips a head. What is the probability that they miss the free throw and then flip tails?

	H	T
Make		
Miss		
Miss		

By finding the area of the small rectangles, the probabilities are:
P(make *and* heads) $= \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$, and P(miss *and* tails) $= \frac{2}{3} \cdot \frac{1}{2} = \frac{2}{6}$

Example 4

Chris owns a coffee cart that he parks outside the downtown courthouse each morning. 65% of his customers are lawyers; the rest are jury members. 60% of Chris's sales include a muffin, 10% include cereal, and the rest are coffee only. What is the probability a lawyer purchases a muffin or cereal?

The probabilities could be represented in an area model as follows:

	lawyer 0.65	jury 0.35
muffin 0.60		
cereal 0.10		
coffee only 0.30		

Probabilities can then be calculated:

	lawyer 0.65	jury 0.35
muffin 0.60	0.39	0.21
cereal 0.10	0.065	0.035
coffee only 0.30	0.195	0.105

The probability a lawyer purchases a muffin or cereal is
$0.39 + 0.065 = 0.455$ or 45.5%.

Example 5

The local ice cream store has choices of plain, sugar, or waffle cones. Their ice cream choices are vanilla, chocolate, bubble gum, or frozen strawberry yogurt. The following toppings are available for the ice cream cones: sprinkles, chocolate pieces, and chopped nuts.

What are all the possible outcomes for a cone *and* one scoop of ice cream *and* a topping? How many outcomes are possible?

Probability tables are useful only when there are two events. In this situation there are three events (cone, flavor, topping), so we will use a probability tree.

There are four possible flavors, each with three possible cones. Then each of those 12 outcomes can have three possible toppings. There are 36 outcomes for the compound event of choosing a flavor, cone, *and* topping.

Note that the list of outcomes, and the total number of outcomes, does not change if we change the order of events. We could just as easily have chosen the cone first.
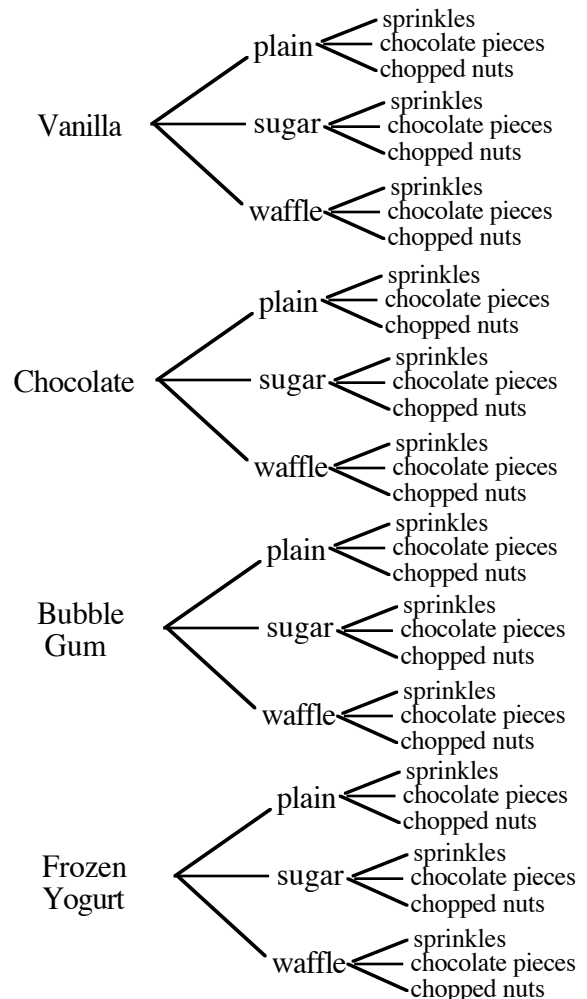
Problems

Use probability tables or tree diagrams to solve these problems.

1. How many different combinations are possible when buying a new bike if the following options are available:

    • mountain bike or road bike

    • black, red, yellow, or blue paint

    • 3–speed, 5–speed, or 10–speed

2. A new truck is available with:

    • standard or automatic transmission

    • 2–wheel or 4–wheel drive

    • regular or king cab

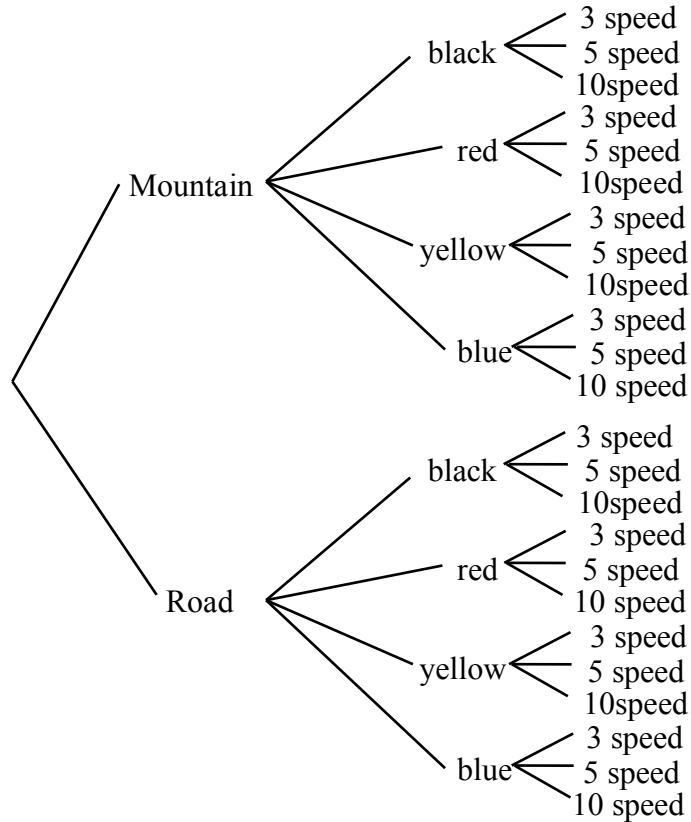   • long or short bed

    How many combinations are possible?

3. A tax assessor categorizes 25% of the homes in how city as having a large backyard, 65% as having a small backyard, and 10% as having no backyard. 30% of the homes have a tile roof, the rest have some other kind of roof. What is the probability a home with a tile roof has a backyard?

4. There is space for only 96 students at University High School to enroll in a "shop" class: 25 students in woodworking, 25 students in metalworking, and the rest in print shop. Three-fourths of the spaces are reserved for seniors, and one-fourth are for juniors. What is the probability that a student enrolled in shop class is a senior in print shop? What is the probability that a student enrolled in shop class is a junior in wood or metal shop?

5. Insurance companies use probabilities to determine the rate they will charge for an insurance policy. In a study of 3000 people that had life insurance policies, an insurance company collected the following data of how old people were when they died, compared to how tall they were. In this study, what was the probability of being tall (over 6ft) and dying young under 50 years old? What was the probability of being tall and dying under 70 years old? What was the probability of being between 50 and 70 years old?

	<50 years old	50–60 yrs old	60–70 yrs old	70–80yrs old	>80 years old
over 6ft tall	30	25	52	82	111
under 6ft tall	270	225	468	738	999

Answers

1. There are 24 possible combinations as shown below.

    Mountain
    - black
      - 3 speed
      - 5 speed
      - 10speed
    - red
      - 3 speed
      - 5 speed
      - 10speed
    - yellow
      - 3 speed
      - 5 speed
      - 10speed
    - blue
      - 3 speed
      - 5 speed
      - 10 speed

    Road
    - black
      - 3 speed
      - 5 speed
      - 10speed
    - red
      - 3 speed
      - 5 speed
      - 10 speed
    - yellow
      - 3 speed
      - 5 speed
      - 10speed
    - blue
      - 3 speed
      - 5 speed
      - 10 speed

2. There are 16 possible combinations as shown below.

    Standard
    - 2 -wheel drive
      - regular cab
        - long bed
        - short bed
      - king cab
        - long bed
        - short bed
    - 4 -wheel drive
      - regular cab
        - long bed
        - short bed
      - king cab
        - long bed
        - short bed

    Automatic
    - 2 -wheel drive
      - regular cab
        - long bed
        - short bed
      - king cab
        - long bed
        - short bed
    - 4 -wheel drive
      - regular cab
        - long bed
        - short bed
      - king cab
        - long bed
        - short bed
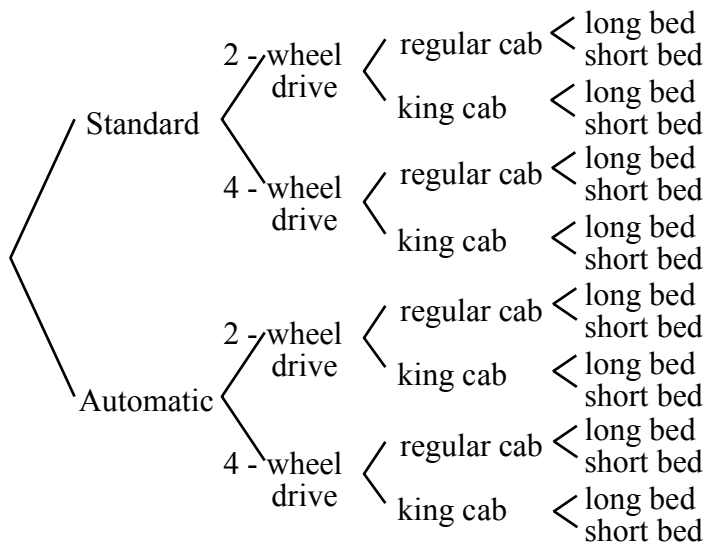
3. The probability is $0.075 + 0.2275 = 0.3025$ or 30.25%.

	large yard 25%	small yard 65%	no yard 10%
tile roof 30%	0.075	0.2275	
other roof 70%			

4. The probability of a senior in print shop is about 0.359%. The probability of a junior in wood or metal shop is $0.065 + 0.065 \approx 0.13$.

	seniors $\frac{3}{4}$	juniors $\frac{1}{4}$	
woodworking $\frac{25}{96}$		≈ 0.065	
metalworking $\frac{25}{96}$		≈ 0.065	
print shop $\frac{46}{96}$	≈ 0.359		

5. The probability of being tall (over 6ft) and dying young under 50 years old is $\frac{30}{3000} = 0.01$. The probability of being tall and dying under 70 years old is $\frac{30+25+52}{3000} \approx 0.036$. The probability of being between 50 and 70 years old is $\frac{25+52+225+468}{3000} \approx 0.257$.

The 5-D process is one method that students can use to solve various types of problems, especially word problems. The D's stand for Describe, Define, Do, Decide, and Declare. When students use the 5-D process, it provides a record of the student's thinking. The patterns in the table lead directly to writing algebraic equations for the word problems.

Writing equations is one of the most important algebra skills students learn. Using the 5-D process helps to make this skill accessible to all students. In order to help students see the relationships in a word problem, we require them to include at least four entries (rows) in their tables. The repetition of the operations is needed to see how the columns are related. After students have had practice using the 5-D process to solve problems, we begin generalizing from the patterns in the table to write an equation that represents the relationships in the problem.

We also believe that writing the answer in a sentence after the table is complete is important because many students forget what the question actually was. The sentence helps the student see the "big picture" and brings closure to the problem.

See the Math Notes box in Lesson 5.3.3 of the *Core Connections, Course 2* text.

Example 1

A box of fruit has three times as many nectarines as grapefruit. Together there are 36 pieces of fruit. How many pieces of each type of fruit are there?

Step 1: **Describe:** Number of nectarines is three times the number of grapefruit.
Number of nectarines plus number of grapefruit equals 36.

Step 2: **Define:** Set up a table with columns. The first column should be the item you know the least about. Choose any easy amount for that column.

<div align="center">

Define

	# of Grapefruit	
Trial 1:	11	

</div>

What else do we need to know?

The number of nectarines, which is three times the number of grapefruit.

<div align="center">

Define

	# of Grapefruit	# of Nectarines
Trial 1:	11	$3(11) = 33$

</div>

Example continues on next page →

Example continued from previous page.

Step 3:   **Do:**   What is the total number of fruit?

	Define		Do	
	# of Grapefruit	# of Nectarines	Total Pieces of Fruit	
Trial 1:	11	33	44	

Step 4:   **Decide:** We need to check the total pieces of fruit based on trial #1 of 11 grapefruit and compare it to the total given in the problem.

	Define		Do	Decide
	# of Grapefruit	# of Nectarines	Total Pieces of Fruit	36?
Trial 1:	11	33	44	too high

Start another trial. Our total was 44; the total needed is 36, so our trial started too high and our next trial should start lower.

	Define		Do	Decide
	# of Grapefruit	# of Nectarines	Total Pieces of Fruit	36?
Trial 1:	11	33	44	too high
Trial 2:	10	30	40	too high

Start another trial. Our total was 40; the total needed is 36, so our trial started too high and our next trial should start still lower.

	Define		Do	Decide
	# of Grapefruit	# of Nectarines	Total Pieces of Fruit	36?
Trial 1:	11	33	44	too high
Trial 2:	10	30	40	too high
Trial 3:	8	24	32	too low

Start another trial. Our total was 32; the total needed is 36, so our trial started too low and our next one should be higher than 8 but lower than 10.
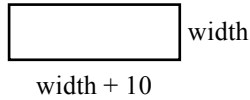
	Define		Do	Decide
	# of Grapefruit	# of Nectarines	Total Pieces of Fruit	36?
Trial 1:	11	33	44	too high
Trial 2:	10	30	40	too high
Trial 3:	8	24	32	too low
Trial 4:	9	27	36	correct

Step 5:   **Declare:** The answer was found. Answer the question in a sentence.
           There are 9 grapefruit and 27 nectarines in the box.

Example 2

The perimeter of a rectangle is 120 feet. If the length of the rectangle is ten feet more than the width, what are the dimensions (length and width) of the rectangle?

Describe/Draw:

width

width + 10

Start with the width because, of the two required answers, it is the one we know the least about. The length is 10 feet more than the width, so add 10 to the first trial.

	Define		**Do**	**Decide**
	Width	Length	Perimeter	120?
Trial 1:	10	20	$(10 + 20) \cdot 2 = 60$	too low

Since the trial of 10 resulted in an answer that is too low, we should increase the number in the next trial. Pay close attention to the result of each trial. Each result helps determine the next trial as you narrow down the possible trials to reach the answer. Note: As students get more experience with using the 5-D process, they learn to make better-educated trials from one step to the next to solve problems quickly or to establish the pattern they need to write an equation.

	Define		**Do**	**Decide**
	Width	Length	Perimeter	120?
Trial 1:	10	20	$(10 + 20) \cdot 2 = 60$	too low
Trial 2:	20	30	100	too low
Trial 3:	30	40	140	too high
Trial 4:	25	35	120	correct

Declare: The dimensions are 25 and 35 feet.

Example 3

Jorge has some dimes and quarters. He has 10 more dimes than quarters and the collection of coins is worth $2.40. How many dimes and quarters does Jorge have?

Note: This type of problem is more difficult than others because the *number* of things asked for is different than their *value*. Separate columns for each part of the problem must be added to the table as shown below. Students often neglect to write the third and fourth columns.

Describe: The number of quarters plus 10 equals the number of dimes.
The total value of the coins is $2.40.

	Define			Do	Decide
# Quarters	# Dimes	Value of Quarters	Value of Dimes	Total Value	$2.40?
Trial 1: 10	20	2.50	2.00	4.50	too high
Trial 2: 8	18	2.00	1.80	3.80	too high
Trial 3: 6	16	1.50	1.60	3.10	too high
Trial 4: 4	14	1.00	1.40	2.40	correct

Declare: Jorge has four quarters and 14 dimes.

HELPFUL QUESTIONS TO ASK YOUR STUDENT

> If your student is having difficulty with a 5-D problem, it may be because he/she does not understand the problem, not because he/she does not understand the 5-D process. Here are some helpful questions to ask when your child does not understand the problem. (These are useful in non-word problem situations, too.)
>
> What are you being asked to find?
>
> What information have you been given?
>
> Is there any unneeded information? If so, what is it?
>
> Is there any necessary information that is missing? If so, what information do you need?

TIPS ABOUT COLUMN TITLES

> 1. You may select any number for the first trial. Ten or the student's age are adequate numbers for the first trial. The result will help you to determine the number to use for the second trial.
>
> 2. Continue establishing columns by asking, "What else do we need to know to determine whether the number we used for our trial is correct or too low or too high?"
>
> 3. Put the answer to one calculation in each column. Students sometimes try to put the answer to several mental calculations in one column. (See the note in Example 3.)

Problems

Solve these problems using the 5-D process. Write each answer in a sentence.

1. A wood board 100 centimeters long is cut into two pieces. One piece is 26 centimeters longer than the other. What are the lengths of the two pieces?

2. Thu is five years older than her brother Tuan. The sum of their ages is 51. What are their ages?

3. Tomas is thinking of a number. If he triples his number and subtracts 13, the result is 305. What is the number that Tomas is thinking about?

4. Two consecutive numbers have a sum of 123. What are the two numbers?

5. Two consecutive even numbers have a sum of 246. What are the numbers?

6. Joe's age is three times Aaron's age and Aaron is six years older than Christina. If the sum of their ages is 149, what is Christina's age? Joe's age? Aaron's age?

7. Farmer Fran has 38 barnyard animals, consisting of only chickens and goats. If these animals have 116 legs, how many of each type of animal are there?

8. A wood board 156 centimeters long is cut into three parts. The two longer parts are the same length and are 15 centimeters longer than the shortest part. How long are the three parts?

9. Juan has 15 coins, all nickels and dimes. This collection of coins is worth 90¢. How many nickels and dimes are there? (Hint: Create separate column titles for, "Number of Nickels," "Value of Nickels," "Number of Dimes," and "Value of Dimes.")

10. Tickets to the school play are $ 5.00 for adults and $ 3.50 for students. If the total value of all the tickets sold was $2517.50 and 100 more students bought tickets than adults, how many adults and students bought tickets?

11. A wood board 250 centimeters long is cut into five pieces: three short ones of equal length and two that are both 15 centimeters longer than the shorter ones. What are the lengths of the boards?

12. Conrad has a collection of three types of coins: nickels, dimes, and quarters. There is an equal amount of nickels and quarters but three times as many dimes. If the entire collection is worth $ 9.60, how many nickels, dimes, and quarters are there?

Answers

1. The lengths of the boards are 37 cm and 63 cm.

2. Thu is 28 years old and her brother is 23 years old.

3. Tomas is thinking of the number 106.

4. The two consecutive numbers are 61 and 62.

5. The two consecutive numbers are 142 and 144.

6. Christine is 25, Aaron is 31, and Joe is 93 years old.

7. Farmer Fran has 20 goats and 18 chickens.

8. The lengths of the boards are 42, 57, and 57 cm.

9. Juan has 12 nickels and 3 dimes.

10. There were 255 adult and 355 student tickets purchased for the play.

11. The lengths of the boards are 44 and 59 cm.

12. Conrad has 16 nickels and quarters and 48 dimes.

At first students used the 5-D Process to solve problems. However, solving complicated problems with the 5-D Process can be time consuming and it may be difficult to find the correct solution if it is not an integer. The patterns developed in the 5-D Process can be generalized by using a variable to write an equation. Once you have an equation for the problem, it is often more efficient to solve the equation than to continue to use the 5-D Process. Most of the problems here will not be complex so that you can practice writing equations using the 5-D Process. The same example problems previously used are used here except they are now extended to writing and solving equations.

Example 1

A box of fruit has three times as many nectarines as grapefruit. Together there are 36 pieces of fruit. How many pieces of each type of fruit are there?

Describe: Number of nectarines is three times the number of grapefruit.
Number of nectarines plus number of grapefruit equals 36.

	Define		**Do**	**Decide**
	# of Grapefruit	# of Nectarines	Total Pieces of Fruit	36?
Trial 1:	11	33	44	too high
Trial 2:	10	30	40	too high

After several trials to establish a pattern in the problem, you can generalize it using a variable. Since we could try any number of grapefruit, use x to represent it. The pattern for the number of oranges is three times the number of grapefruit, or $3x$. The total pieces of fruit is the sum of column one and column two, so our table becomes:

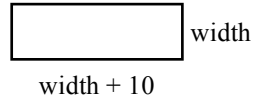|	**Define**		**Do**	**Decide**
	# of Grapefruit	# of Nectarines	Total Pieces of Fruit	36?
	x	$3x$	$x + 3x$	$= 36$

Since we want the total to agree with the check, our equation is $x + 3x = 36$. Simplifying this yields $4x = 36$, so $x = 9$ (grapefruit) and then $3x = 27$ (nectarines).

Declare: There are 9 grapefruit and 27 nectarines.

Example 2

The perimeter of a rectangle is 120 feet. If the length of the rectangle is 10 feet more than the width, what are the dimensions (length and width) of the rectangle?

Describe/Draw:

width + 10 / width (rectangle)

width

width + 10

	Define		**Do**	**Decide**
	Width	Length	Perimeter	120?
Trial 1:	10	25	$(10 + 25) \cdot 2 = 70$	too low
Trial 2:	20	30	100	too low

Again, since we could guess any width, we labeled this column x. The pattern for the second column is that it is 10 more than the first: $x + 10$. The perimeter is found by multiplying the sum of the width and length by 2. Our table now becomes:

	Define		**Do**	**Decide**
	Width	Length	Perimeter	120?
	x	$x + 10$	$(x + x + 10) \cdot 2$	$= 120$

Solving the equation:

$$(x + x + 10) \cdot 2 = 120$$
$$2x + 2x + 20 = 120$$
$$4x + 20 = 120$$
$$4x = 100 \quad \text{So } x = 25 \text{ (width) and } x + 10 = 35 \text{ (length).}$$

Declare: The width is 25 feet and the length is 35 feet.

Example 3

Jorge has some dimes and quarters. He has 10 more dimes than quarters and the collection of coins is worth $2.40. How many dimes and quarters does Jorge have?

Describe: The number of quarters plus 10 equals the number of dimes.
The total value of the coins is $2.40.

	Define				**Do**	**Decide**
	Quarters	Dimes	Value of Quarters	Value of Dimes	Total Value	$2.40?
Trial 1:	10	20	2.50	2.00	4.50	too high
Trial 2:	8	18	2.00	1.80	3.80	too high
	x	$x + 10$	$0.25x$	$0.10(x + 10)$	$0.25x + 0.10(x + 10)$	

Since you need to know both the number of coins and their value, the equation is more complicated. The number of quarters becomes x, but then in the table the Value of Quarters column is $0.25x$. Thus the number of dimes is $x + 10$, but the value of dimes is $0.10(x + 10)$. Finally, to find the numbers, the equation becomes $0.25x + 0.10(x + 10) = 2.40$.

Solving the equation:
$$0.25x + 0.10x + 1.00 = 2.40$$
$$0.35x + 1.00 = 2.40$$
$$0.35x = 1.40$$
$$x = 4.00$$

Declare: There are 4 quarters worth $1.00 and 14 dimes worth $1.40 for a total value of $2.40.

Problems

Start the problems using the 5-D Process. Then write an equation. Solve the equation.

1. A wood board 100 centimeters long is cut into two pieces. One piece is 26 centimeters longer than the other. What are the lengths of the two pieces?

2. Thu is five years older than her brother Tuan. The sum of their ages is 51. What are their ages?

3. Tomás is thinking of a number. If he triples his number and subtracts 13, the result is 305. Of what number is Tomás thinking?

4. Two consecutive numbers have a sum of 123. What are the two numbers?

5. Two consecutive even numbers have a sum of 246. What are the numbers?

6. Joe's age is three times Aaron's age and Aaron is six years older than Christina. If the sum of their ages is 149, what is Christina's age? Joe's age? Aaron's age?

Core Connections, Course 2

7. Farmer Fran has 38 barnyard animals, consisting of only chickens and goats. If these animals have 116 legs, how many of each type of animal are there?

8. A wood board 156 centimeters long is cut into three parts. The two longer parts are the same length and are 15 centimeters longer than the shortest part. How long are the three parts?

9. Juan has 15 coins, all nickels and dimes. This collection of coins is worth 90¢. How many nickels and dimes are there? (Hint: Create separate column titles for, "Number of Nickels," "Value of Nickels," "Number of Dimes," and "Value of Dimes.")

10. Tickets to the school play are $ 5.00 for adults and $ 3.50 for students. If the total value of all the tickets sold was $2517.50 and 100 more students bought tickets than adults, how many adults and students bought tickets?

11. A wood board 250 centimeters long is cut into five pieces: three short ones of equal length and two that are both 15 centimeters longer than the shorter ones. What are the lengths of the boards?

12. Conrad has a collection of three types of coins: nickels, dimes, and quarters. There is an equal amount of nickels and quarters but three times as many dimes. If the entire collection is worth $ 9.60, how many nickels, dimes, and quarters are there?

Answers (Equations may vary.)

1. $x + (x + 26) = 100$
   The lengths of the boards are 37 cm and 63 cm.

2. $x + (x + 5) = 51$
   Thu is 28 years old and her brother is 23 years old.

3. $3x - 13 = 305$
   Tomás is thinking of the number 106.

4. $x + (x + 1) = 123$
   The two consecutive numbers are 61 and 62.

5. $x + (x + 2) = 246$
   The two consecutive numbers are 142 and 144.

6. $x + (x + 6) + 3(x + 6) = 149$
   Christine is 25, Aaron is 31, and Joe is 93 years old.

7. $2x + 4(38 - x) = 116$
   Farmer Fran has 20 goats and 18 chickens.

8. $x + (x + 15) + (x + 15) = 156$
   The lengths of the boards are 42, 57, and 57 cm.

9. $0.05x + 0.10(15 - x) = 0.90$
   Juan has 12 nickels and 3 dimes.

10. $\$5x + \$3.50(x + 100) = 2517.50$
    There were 255 adult and 355 student tickets purchased for the play.

11. $3x + 2(x + 15) = 250$
    The lengths of the boards are 44 and 59 cm.

12. $0.05x + 0.25x + 0.10(3x) = 9.60$
    Conrad has 16 quarters, 16 nickels, and 48 dimes.